A PERTURBATION THEORY FOR DIELECTRIC AND OPTICAL WAVEGUIDES WITH APPLICATION TO THE LAUNCHING OF SURFACE MODES

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A perturbation technique is employed for dielectric waveguides with a small dielectric difference between the guide and its surrounding medium, resulting in an analytically simple, self-consistent, theory for surface modes. The field equations are shown to manifest bounded waveguide simplicity. In particular the transverse electric and magnetic fields are related by a constant and possess the orthogonality of a metal waveguide. Furthermore, an analytic expression is presented for the eigenvalues. Although the analysis is based on small dielectric differences, it is shown to be adequate when the inside dielectric is as large as twice the outside. The results of the perturbation analysis are applied to the excitation of a semi-infinite dielectric rod excited by a uniform field.

In general the equations for the eigenfunctions and eigenvalues of a cylindrical dielectric waveguide are complicated expressions. Several authors [1,2] have noted simplifications, principally determined numerically, for the situation of small dielectric difference between the rod and its surrounding medium. (The condition of small dielectric differences is common with practical devices [2,4]). In this paper a perturbation technique is used to formalise and extend the small dielectric

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difference approximation, resulting in an analytically simple, self consistent, theory.

The results of the perturbation analysis have been applied by the author to the study of mode propagation along dielectric tapers [3]. In this paper application is made to the launching of surface modes by a uniform excitation.

The essential results of the perturbation theory are presented as follows: It is assumed that $\delta << 1$

$$\delta = 1 - (\varepsilon_2/\varepsilon_1) \qquad \ldots(1)$$

ε_1, ε_2 is the dielectric constant of the rod and of the surrounding medium respectively. Then, by neglecting terms of order δ, the following simplifications are made possible. Orthogonality of the transverse modal vector fields $\bar{e}_p$, $\bar{h}_p$ is given as

$$\int \bar{e}_p \times \bar{h}_q^* \cdot \bar{da} = \sqrt{\varepsilon_1/\mu} \;\; \bar{e}_p \cdot \bar{e}_q^* \, da \qquad (a)$$

$$= \sqrt{\mu/\varepsilon_1} \int \bar{h}_p \cdot \bar{h}_q^* \, da \qquad (b) \ldots(2)$$

$$= \delta_{pq} \qquad (c)$$

(Integration is extended over the entire transverse plane. The complex form of orthogonality restricts the analysis to lossless media.)

The orthonormal fields $\bar{e}_p$, $\bar{h}_p$ are defined as

$$\bar{h}_p = \sqrt{\varepsilon_1/\mu} \;\; (\hat{z} \times \bar{e}_p) = \underline{h}_p/\sqrt{\psi} \qquad (a)$$

$$\bar{e}_p = \underline{e}_p/\sqrt{\psi} \qquad (b) \ldots(3)$$

where,

$$\underline{e}_R = \pm f_\ell(R)\ \sin(\ell\phi + \alpha_p)\ e^{j(\omega t-\beta_p Z)} \qquad \text{(a)}$$

$$= -f_o(R)\ e^{j(\omega t-\beta_p Z)} \qquad (\ell = 0,\ \text{TM}) \qquad \ldots(4)$$

$$\underline{e}_\phi = f_\ell(R)\ \cos(\ell\phi + \alpha_p)\ e^{j(\omega t-\beta_p Z)} \qquad \text{(b)}$$

$$= f_o(R)\ e^{j(\omega t -\beta_p Z)} \qquad (\ell = 0,\ \text{TE})$$

where, in the above equations and throughout the paper, the upper sign is taken for the $HE_{\ell M}$ modes and the lower for the $EH_{\ell M}$ modes. (The subscripts p are dropped throughout the remainder of the text for simplicity of notation).

$$f\ (R) = J_{\ell\mp 1}(UR)\ /\ J_{\ell\mp 1} \qquad R \le 1$$

$$= K_{\ell\mp 1}(WR)\ /\ K_{\ell\mp 1} \qquad R \ge 1 \qquad \ldots(5)$$

$J_\ell = J_\ell(U)$ Bessel functions of order ℓ; and $K_\ell = K_\ell(W)$ modified Hankel function of order ℓ.

$$\psi = \int \bar{\underline{e}}_p \times \bar{\underline{h}}^*_p \cdot \overline{da}$$

$$= \rho^2 \pi \sqrt{\varepsilon_1/\mu}\ (V/U)^2\ K_\ell K_{\ell\mp 2}\ /\ K^2_{\ell\mp 1} \qquad \ldots(6)$$

$$V = (\rho\omega)^2\ \varepsilon_1 \mu\delta \qquad \ldots(7)$$

(V is held constant in the perturbation procedure; it is not considered as order δ.)

$$\beta = (V/\sqrt{\delta})\ (1 - \delta U^2/2V^2) \qquad \ldots(8)$$

$$R = r/\rho \qquad \ldots(9)$$

$$Z = z/\rho \qquad \ldots(10)$$

U and W are the normalised eigenvalues for $R < 1$, $R > 1$ respectively; ρ is the radius of the rod. $\hat{z}$ is the unit vector in the z direction, ϕ is the azimuthal angle, μ is the permeability of the media and α_p is the phase constant.

For all mode types the eigenvalue equation is

$$(UJ_\ell / J_{\ell \mp 1}) = \pm (WK_\ell / K_{\ell \mp 1}) \qquad \ldots(11)$$

U, V and W are related as

$$V^2 = U^2 + W^2 \qquad \ldots(12)$$

Although (11) requires a numerical solution, a rather useful analytic representation can be derived from it. Asymptotically, above cut-off

$$U(V) \sim U(\infty)\, e^{-1/V} \qquad \ldots(13)$$

where

$$U(\infty) = \text{roots of } J_{\ell \mp 1} \qquad \ldots(14)$$

$$= 2.405\ (HE_{11}),\ 3.832\ TM_{01},\ TE_{01},\ HE_{21},\ \text{etc.}$$

A comparison of the approximate expression for U given by (13) with that of the numerical solution of (11) is displayed in figure 1 and is observed to be in excellent agreement except very close to cut-off. The numerical results of (11) compared to the exact U are shown to have an error of less than 1% for $\delta \leq .2$, and less than 10% at $\delta = .5$ (inside dielectric twice outside) Ref. [1].

Application to Surface Mode Launching

Consider a semi-infinite dielectric rod extending from $z = 0$ to ∞, excited by a uniform field of zero amplitude except over the radius ρ.

Such a field can be realised with either a horn or lazer. The wave is incident with the z axis at an angle θ and is assumed to transmit unity power. For the condition of $\delta \ll 1$ and $\theta \ll 1$ it is shown that at:

(a) Normal incidence ($\theta = 0$)

Only the HE_{1M} modes are excited with power [5]

$$P = (2W/VU)^2 \qquad \ldots(15)$$

(b) Arbitrary incidence

Depending on V all modes are excited with one exception. Either the TM or the TE mode set is launched; interchanging the electric and magnetic incident wave excites the other. The condition for maximum power to be launched in a particular mode is given approximately when

$$(V/\sqrt{\delta}) = U_p \qquad \ldots(16)$$

Then the power of mode p is

$$P_p = J_\ell(\theta V/\sqrt{\delta}) \qquad \ldots(17)$$

Conditions (16) and (17) are not valid for the HE_{11} mode which has a maximum at $\theta = 0$ given by (15).

References

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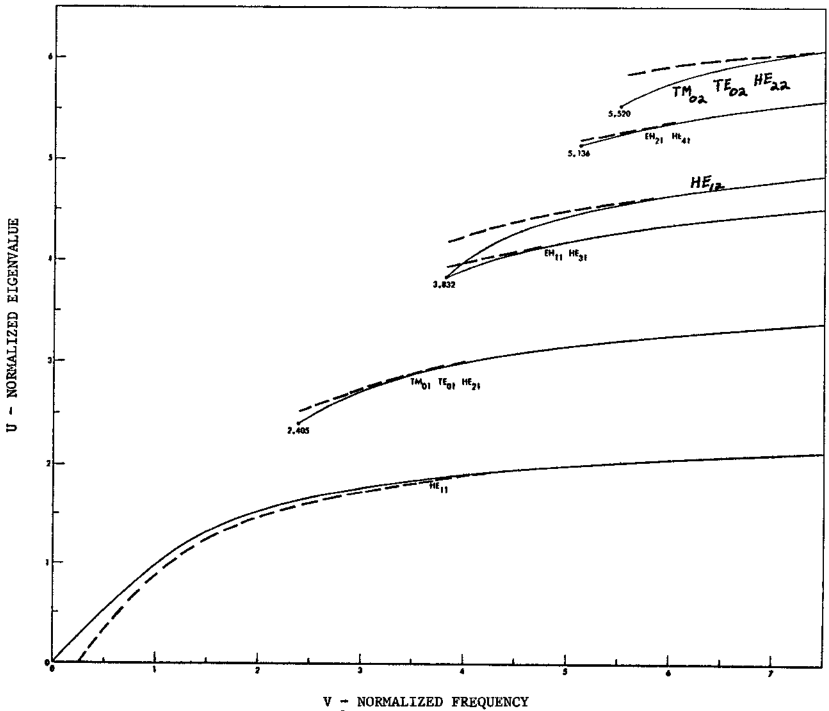

Figure 1. Normalised frequency, $V^2 = (2\pi\rho/\lambda)^2\delta$, VRS the normalised eigenvalue U for $\delta = 0$. The dashed curve represents an approximate solution given by $U_q(V) = U_q(\infty)^{-1/V}$